

# UNIT-1

## SIMPLE STRESSES & STRAINS

(Ref) Strength of material - Dr. R.K. Bansal

Strength:- It is defined as the resistance against failure is called strength. It is a material property.

Stiffness:- Resistance against deformation is stiffness. This is a secondary design property. The resistance by which material of the body opposes the deformation is known as "strength of material" within a certain limit (i.e. elastic limit).

Stress:- The force of resistance per unit area, offered by a body against deformation is known as stress. The external force acting on the body is called the load or force.

mathematically stress written as

$$\sigma = \frac{P}{A}$$

where  $\sigma$  = stress (also called intensity of stress)

$P$  = External force of load

$A$  = cross-sectional area

Units of Stress:- The unit of stress is depends on load (unit are

i.e.  $\rightarrow$   $\text{N/mm}^2$  (or)  $\text{KN/m}^2$  unit-1 pg 1/3

the various types of stress may be classified as -

### Stress

#### Normal

#### Shear

##### Direct

##### Indirect

##### Direct

##### Indirect

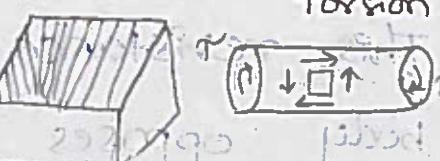
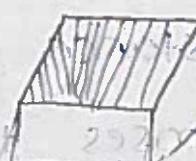
##### Tension

##### Bending

##### Tangential force

##### Torsion

##### Compression



The stresses may be normal stress (or)

a shear stress the normal stress is further divided into tensile and compressive stress.

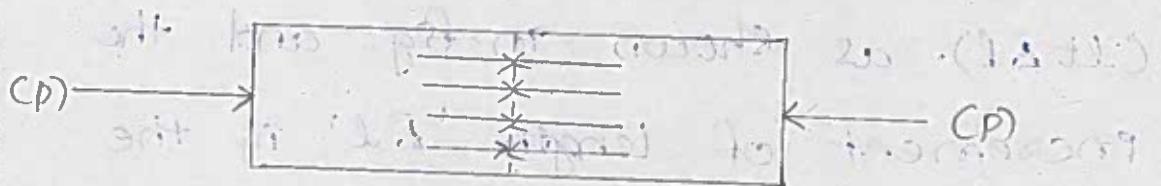
Tensile Stress:- The stress induced in a body, when subjected to two equal and opposite pulls, as shown in fig. as a result of which there is an increase in length. It is known as tensile stress.



Compressive Stress:- the stress induced in a body, when subjected to two equal and opposite pushes as shown in fig. as a result of which there is a decrease in length of the body, is

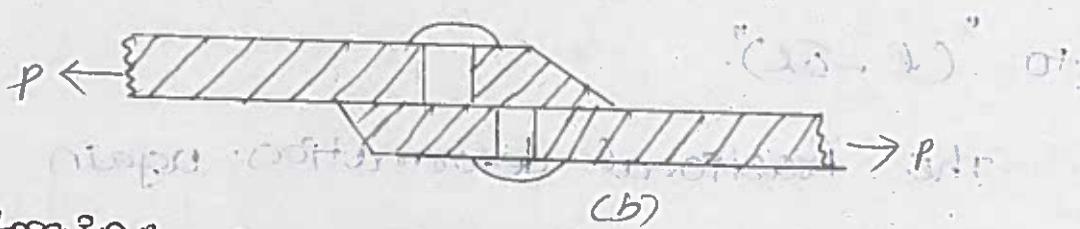
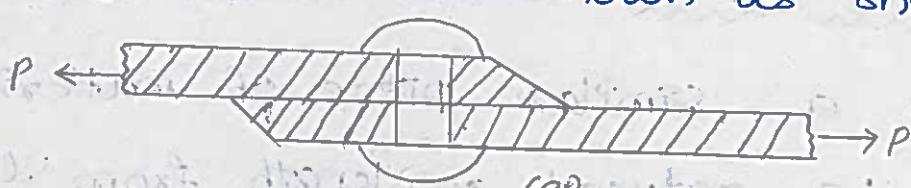
②

is known as compressive stress



compressive load

Shear stress:- The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as shown in fig. as a result of which the body tends to shear off across the section is known as shear stress. The corresponding strain is known as shear strain.



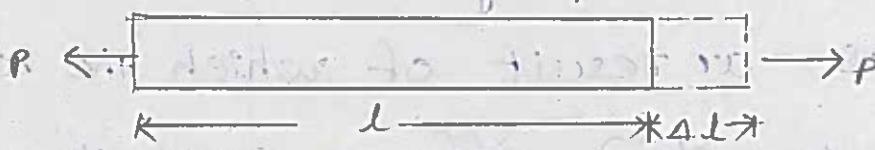
Strain:- Any element in a material subjected to stress is said to be strained.

The strain ( $\epsilon$ ) is the deformation produced by stress. The various types of strain are explained below.

Tensile strain:- A piece of material with uniform cross-section subjected to a uniform axial tensile stress will

increase in its length from ' $l$ ' to  $(l + \Delta l)$ . as shown in fig. and the increment of length ' $\Delta l$ ' is the actual deformation of the material the fractional deformation (or) the tensile strain is given by.

$$e_t = \frac{\Delta l}{l}$$



Compressive strain :- Under compressive forces, a similar piece of material would be reduced in length from ' $l$ ' to " $(l - \Delta l)$ ".

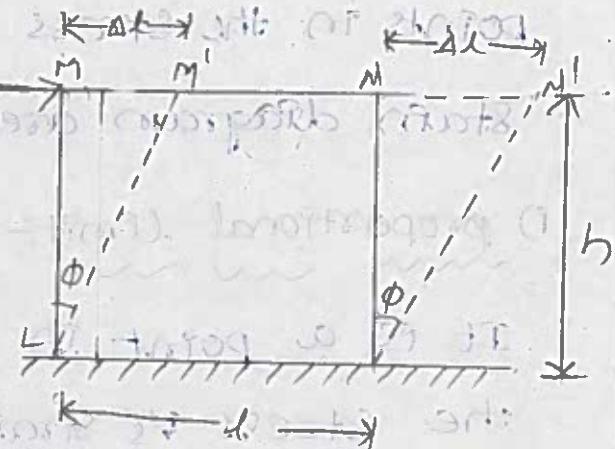
The fractional deformation again gives the strain.

$$e_c = \frac{\Delta l}{l}$$



Shear strain :- In case of a shearing load, a shear strain will be produced which is measured by the angle through

(3) which the body distorts fig shows the rectangular above block LMNR fixed at one face and subjected to force F. After application of force, it distorts, through an angle ' $\phi$ ' and occupies new position LM'N'P. The shear strain ( $e_s$ ) is given by



$$e_s = \frac{NN'}{NP} = \frac{\Delta l}{h} = \tan \phi$$

Volumetric strain :- It is defined as the ratio between change in volume and original volume of the body and is denoted by  $e_v$ .

$$\therefore e_v = \frac{\text{Change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

Hooke's law :- Robert Hooke discovered experimentally that, within the elastic limit, stress varies directly as strain.  
i.e. stress  $\propto$  strain.

Stress & strain diagram for mild steel:

It is a destructive testing which is carried out in "universal testing machine" (utm).

The significant points in the stress

strain diagram are.

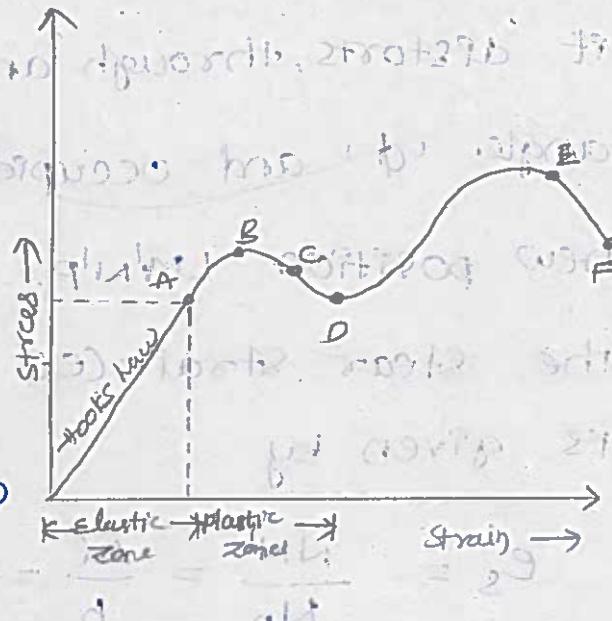
i) proportional limit:-

It is a point in the stress vs strain diagram upto which the diagram is perfectly straight line. i.e from

point 'O' to 'A'. indicates proportional limit. which may be considered as Hooke's law:

vii) Elastic limit(B):- It is point above proportional limit but very close to it. upto elastic limit, property of elasticity of the material is exhibited, but Hooke's law is not obeyed upto this point. In fig 'B' indicates the elastic limit.

iii) upper yield point(C):- 'C' is the upper yield point in the portion Bc, the



④

metal shows an appreciable strain without further increase of stress, and the strain is not fully recovered when the external load is removed from the metal.

iv) lower yield point (D): - 'D' is the lower yield point yielding strain in the portion CD and there is a drop of stress at the point 'D' as soon as yielding starts at C.

v) point of ultimate stress (E): - 'E' is the point of ultimate stress. In the portion DE, there is increase of stress and strain, though not proportionately till point E, is reached when a neck occurs, and the cross sectional area of the specimen being tested, is abruptly reduced. Stress just before formation of such neck is called ultimate stress.

vi) Breaking point (F): - 'F' is the breaking point. At the breaking point actual rupture of the metal occurs. Since after reaching ultimate stress, cross sectional area of the metal is reduced, the stress at

which finally the specimen fails is called breaking point. at this stage strain is 20 to 25%.

working stress is it is the maximum stress allowed to be set up in a material in actual practice.

working stress is much less than ultimate stress it is obtained by dividing the ultimate stress by a number, called factor of safety. factor of safety is such selected that the working stress is less than or equal to proportional limit.

factor of safety:- it is defined as the ratio of ultimate stress to the working stress.

mathematically, it is written as

$$\text{factor of safety} = \frac{\text{ultimate stress}}{\text{permissible stress (or) working stress.}}$$

→ for stress = 1.85

→ for concrete = 3.

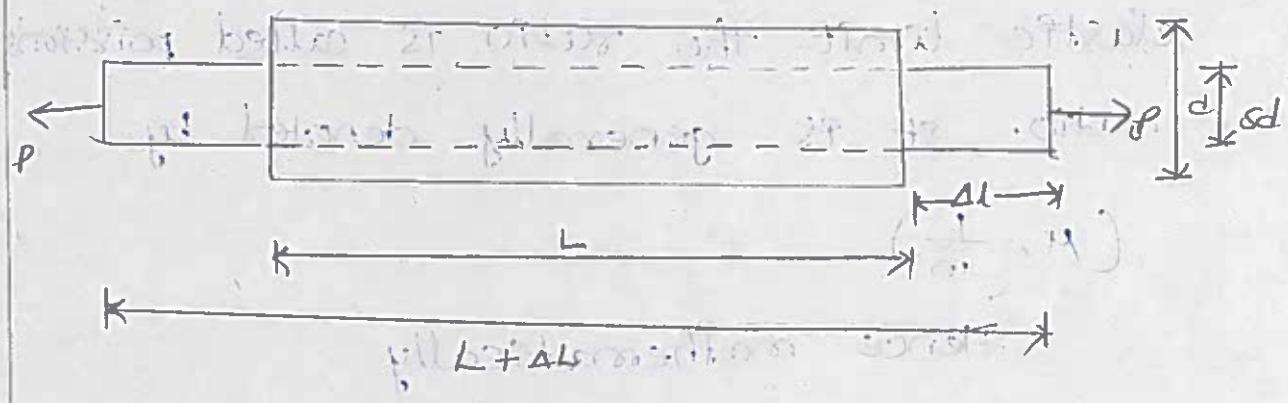
(5) Elastic constants :- Young's modulus, Rigidity modulus, Bulk modulus, Poisson's ratio

Its are the elastic constants.

Longitudinal strain :- When a body is subjected to an axial tensile load, there is an increase in the length of the body. But the same time there is a decrease in other dimensions of the body of right angle to the line of the applied load. Thus the body is having axial deformation and also deformation at right angles to the line of action of the applied load (i.e. lateral deformation).

The longitudinal strain is also defined, as the deformation of the body per unit length in the direction of the applied load.

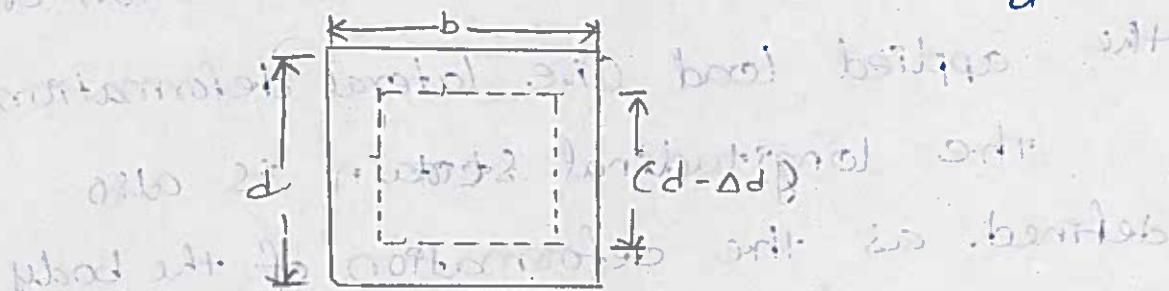
$$\therefore \text{longitudinal strain} = \frac{\Delta L}{L}$$



Lateral Strain— The strain at right angle to the direction of applied load is known as lateral strain. Let a rectangular bar of length  $L$ , breadth  $b$  and depth ' $d$ ' is subjected to an axial tensile load ' $P$ ' as shown in fig. the length of the bar will increase while the breadth and depth will decrease.

$$\therefore \text{Longitudinal strain} = \frac{\Delta L}{L}$$

$$\text{Lateral strain} = \frac{\Delta b}{b} \text{ or } \frac{\Delta d}{d}$$



Poisson's Ratio— The ratio of lateral strain to longitudinal strain is a constant for a given material when the material is stressed within the elastic limit. The ratio is called poisson's ratio. It is generally denoted by

$$(M, \perp)$$

Hence mathematically

$$\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

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⑥

$$\text{Lateral strain} = -[\mu \times \text{longitudinal strain}]$$

-  $\mu$  indicates as lateral strain is opposite in sign to longitudinal strain, hence algebraically, it is written as.

$$\text{lateral strain} = -\mu \times \text{longitudinal strain}$$

Volumetric strain :- It is defined as the ratio between change in volume and original volume of the body and is denoted by  $e_v$ .

$$e_v = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\delta v}{v}$$

where  $\delta v$  = change in volume  
 $v$  = original volume.

Relationship b/w stress and strain :-

The relationship between stress and strain can be explained as below.

i) 1-Dimensional system :- The relationship between stress and strain for a unidirectional stress (for one direction only).

is given by Hooke's law. and it is expressed by ' $E$ ' and is known as modulus of

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elasticity (or) young's modulus.

$$E = \frac{\text{Normal stress}}{\text{Corresponding strain}} = \frac{\sigma}{\epsilon}$$

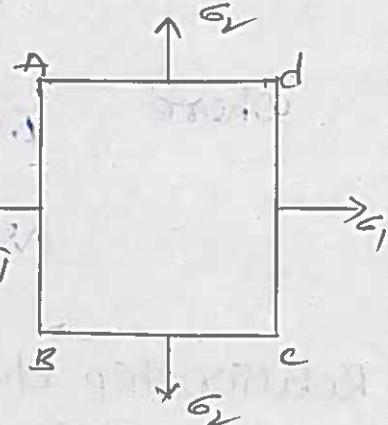
$$\boxed{E = \frac{\sigma}{\epsilon}}$$

i) 2-Dimensional system:— consider a two-dimensional figure ABCD, subjected to two mutually perpendicular stresses  $\sigma_1$  and  $\sigma_2$ .

Now total strain in the direction of

'x' due to stresses  $\sigma_1$  and  $\sigma_2$

$$= \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$



Similarly total strain in the direction of 'y' due to stresses  $\sigma_1$  and  $\sigma_2$

$$= \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

$$\therefore e_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$e_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

(2)

iii) 3-Dimensional system fig

shows a three dimensional

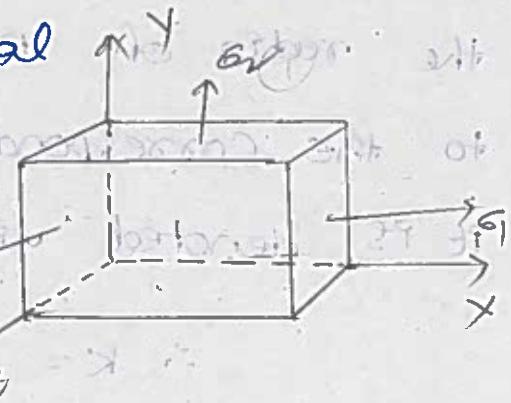
body subjected to three

orthogonal normal stresses

$\sigma_1, \sigma_2, \sigma_3$  acting in the

direction of  $x, y, z$

respectively. Let  $e_1, e_2$  &  $e_3$  are total strains in the direction of  $x, y, z$  respectively, then



$$e_1 = \frac{\sigma_1}{E} - \nu \frac{\sigma_2}{E} - \nu \frac{\sigma_3}{E}$$

$$e_2 = \frac{\sigma_2}{E} - \nu \frac{\sigma_3}{E} - \nu \frac{\sigma_1}{E}$$

$$e_3 = \frac{\sigma_3}{E} - \nu \frac{\sigma_1}{E} - \nu \frac{\sigma_2}{E}$$

The above three equations gives the stress and strain relationship for the three orthogonal normal stress system.

Elastic modulus constants:-

Young's modulus (E): It is the ratio between

tensile stress and tensile strain (or) compressive

stress and compressive strain it is denoted

by E. It is the same as modulus of elasticity.

$$E = \frac{T}{c} \Rightarrow \left[ \frac{\sigma_t}{\epsilon_t} \text{ (or)} \frac{\sigma_c}{\epsilon_c} \right]$$

i) Bulk modulus ( $K$ ) :- It may be defined as the ratio of normal ( $\sigma_x$ ) direct stress to the corresponding volumetric strain. It is denoted with  $K$ .

$$\therefore K = \frac{\text{Direct Stress}}{\text{volumetric strain}} = \frac{\sigma}{\left(\frac{\delta v}{v}\right)}$$

ii) modulus of rigidity ( $C$ ) :- It is defined as the ratio of shear stress ( $\tau$ ) to shear strain and is denoted by  $C$ . It is called shear modulus of elasticity.

$$\boxed{\therefore C = \frac{\tau}{\epsilon_s}}$$

→ Relationship between Young's modulus ' $E$ ' and modulus of rigidity ' $C$ ' :-

Fig. shows the LMSST is. a solid a,b,c subjected to a shearing force  $F$  let  $\tau$  be the shear stress produced in the face MS and LT due to this shearing force. the complementary shear stress consequently produced in the faces ML and ST.

The cube is distorted to LM'S'T and as such, the edge moves to M' and S to S' and the diagonal LS to L'S'

(4)

$$\therefore \text{shear strain } \gamma = \frac{\delta s}{s t}$$

we know that shear strain  $= \frac{\gamma}{c}$

$$\boxed{\therefore \frac{\delta s}{s t} = \frac{\gamma}{c}} \quad - (i)$$

(10)

on the diagonal  $Ls'$  draw a perpendicular  $SN$  from  $S$ .

$$\therefore \text{Now diagonal strain} = \frac{\Delta l}{l} = \frac{Ns'}{LN} = \frac{Ns'}{Ls} \quad - (ii)$$

angle of deformation is very small we can assume  $\angle s'N = 45^\circ$ . hence  $Ns' = ss'$

$$Ns' = ss' \cos 45^\circ = \frac{ss'}{\sqrt{2}}$$

$\therefore Ls'^t$  is assumed to be equal to  $LLst$

since  $ss'$  is very small.

$$\begin{aligned} Ls &= \sqrt{s t^2 + s t^2} \\ &= \sqrt{2 s t^2} \end{aligned}$$

$$\therefore Ls = st \times \sqrt{2}$$

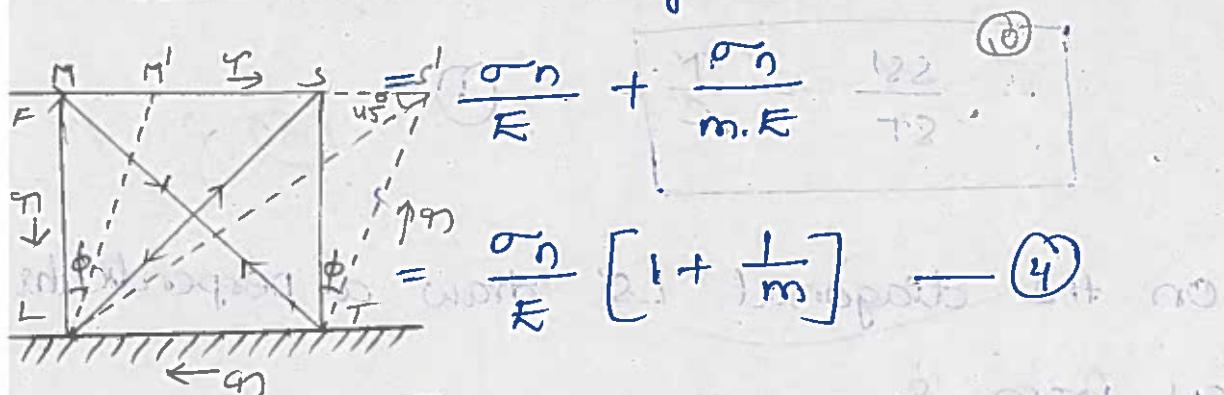
putting the value of  $Ls$  in (ii) we get

$$\text{Diagonal strain} = \frac{ss'}{\sqrt{2} \times st \times \sqrt{2}} = \frac{ss'}{2 \cdot st}$$

$$\text{But } \frac{ss'}{st} = \frac{\gamma}{c}$$

$$\boxed{\therefore \text{Diagonal strain} = \frac{\gamma}{2c} = \frac{\sigma_n}{2e}} \quad - (iii)$$

where  $\sigma_n$  is the normal stress due to shear stress  $\tau$ . the net strain in the direction of diagonal is



$$\frac{\sigma_n}{E} + \frac{\sigma_n}{m.E} = \frac{\sigma_n}{E} \left[ 1 + \frac{1}{m} \right] \quad \textcircled{4}$$

Comparing equations  $\textcircled{3}$  &  $\textcircled{4}$  we get

$$\frac{\sigma_n}{2c} = \frac{\sigma_n}{E} \left[ 1 + \frac{1}{m} \right]$$

$$\text{i.e } E = 2c \left[ 1 + \frac{1}{m} \right]$$

Relation between Young's modulus ( $E$ ) and Bulk modulus ( $K$ ) :-

If the solid cube is subjected to normal compressive stress ( $\sigma_n$ ) on all the faces, the direct strain in each axis =  $\frac{\sigma_n}{E}$  (compressive) & lateral strain in other axis =  $\frac{\sigma_n}{mE}$  (tensile)

$\therefore$  Net compressive strain in each axis

$$= \frac{\sigma_n}{E} - \frac{\sigma_n}{mE} = \frac{\sigma_n}{mE} \left[ 1 - \frac{2}{m} \right]$$

volumetric strain ( $e_v$ ) in this case will be

$$e_v = 3 \times \text{linear strain}$$

$$= 3 \times \frac{\sigma_n}{E} \left[ 1 - \frac{2}{m} \right]$$

$$\text{But } \epsilon_v = \frac{\sigma_n}{k}$$

$$\therefore \frac{\sigma_n}{k} = \frac{3\sigma_n}{E} \left[ 1 - \frac{2}{m} \right] \text{ (eqn)}$$

$$E = 3k \left[ 1 + \frac{2}{m} \right]$$

Relation between  $E, C$  and  $K$ :

The relation between  $E, C$  &  $K$  can be established by eliminating ' $m$ ' from the equations.

$$E = 2C \left[ 1 + \frac{1}{m} \right] \quad \text{and} \quad E = 3K \left[ 1 - \frac{2}{m} \right]$$

$\therefore$  from the above equation ① we can written as

$$E = 2C \left[ 1 + \frac{1}{m} \right]$$

$$\frac{E - 2C}{2C} = \frac{1}{m}$$

$$m = \frac{2C}{E - 2C}$$

Substitute ' $m$ ' value in ② equation:

$$E = 3K \left[ 1 - \frac{3}{2C(E - 2C)} \right]$$

$$E = 3K \left[ 1 - \frac{C(E - 2C)}{C} \right]$$

$$\frac{E}{3K} = \frac{C - E + 2C}{C}$$

$$\frac{E}{3K} = \frac{3C - E}{C}$$

$$\frac{E}{3k} + \frac{E}{c} = 3$$

(a)  $Ec + 3kE = 9kc$

(or)  $E(3k+c) = 9kc$

(or)

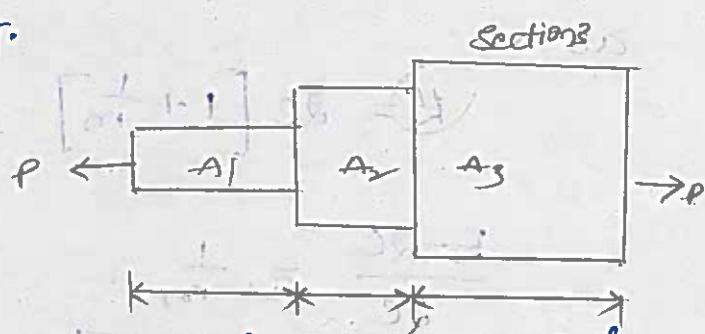
$$E = \frac{9kc}{3k+c}$$

It is the relationship b/w E, c, k.

Analysis of Bars of varying sections:-

A bar of different lengths and of different diameters (and hence of different cross-sectional areas).

as shown in fig.



Let this bar is subjected to an axial load P. Through each section is subjected to the same axial load. Yet the stresses, strains and change in lengths will be different. The total change in length will be obtained by adding the changes in length of individual section.

Let

P = Axial load acting on the bar.

(10)  $L_1$  = length of section 1

$A_1$  = cross-sectional area of section 1

$L_2, A_2$  = length and cross-sectional area of section 2.

$L_3, A_3$  = length and cross-sectional area of section 3.

$E$  = Young's modulus for the bar

then stresses for the section 1.

$$\sigma_1 = \frac{\text{load}}{\text{Area of section 1}} = \frac{P}{A_1}$$

similarly stresses for the section 2 and 3 are given as

$$\sigma_2 = \frac{P}{A_2} \text{ and } \sigma_3 = \frac{P}{A_3}$$

the corresponding strains for different sections are obtained as

$$e_1 = \frac{\sigma_1}{E} = \frac{P}{A_1 E} \quad \left[ \because \sigma_1 = \frac{P}{A_1} \right]$$

similarly  $e_2 = \frac{\sigma_2}{E} = \frac{P}{A_2 E}$

strain in section 1 =  $\frac{\text{change in length of section 1}}{\text{length of section 1}}$

$$\therefore e_1 = \frac{\Delta L_1}{L_1}$$

$\therefore$  change in length of section 1

$$dL_1 = e_1 L_1$$

Substitute  $e_1$  in above equation

$$dL_1 = \frac{PL_1}{A_1 E}$$

$$[\because e_1 = \frac{P}{A_1 E}]$$

Similarly the change in length of section 2 & 3 are given as

$$dL_2 = \frac{PL_2}{A_2 E}$$

$$dL_3 = \frac{PL_3}{A_3 E}$$

$\therefore$  total change in the length of the bar

$$dL = dL_1 + dL_2 + dL_3$$

$$dL = \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E}$$

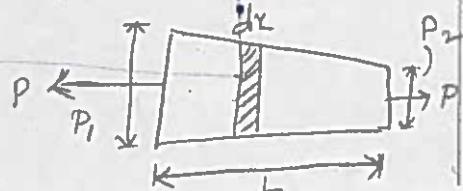
$$dL = \frac{P}{E} \left[ \frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right] \rightarrow \text{used when } E \text{ is same for different sections}$$

$$dL = P \left[ \frac{L_1}{E_1 A_1} + \frac{L_2}{E_2 A_2} + \frac{L_3}{E_3 A_3} \right]$$

$\rightarrow$  used when  $E$  is different for different sections

Note: Change in length for circular rod

$$dL = \frac{4PL}{\pi E \cdot D^2}$$



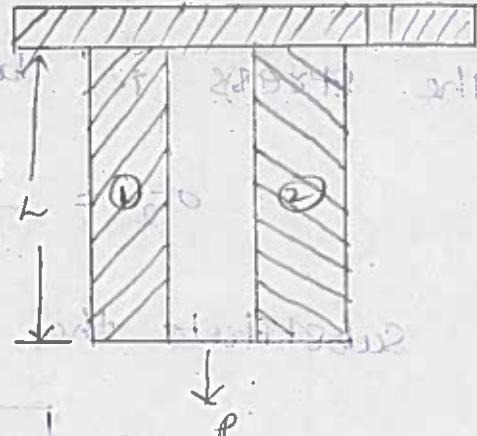
(iv)

Change in length for rectangular Bar (Tapered)

$$\Delta L = \frac{PL}{Ect(a-b)} \log_e \frac{a}{b}$$

Analysis of Bars of Composite Sections:-

A bar is made up of two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for extension (or) compression when subjected to an axial tensile (or) compressive loads, is called a composite bar. Fig shows the composite bar made up of two different materials.

let  $P$  = Total load on the composite bar $L$  = Length of composite bar $A_1$  = Area of cross-section of bar 1 $A_2$  = Area of cross-section of bar 2 $E_1$  &  $E_2$  = Young's modulus for bar 1 & 2 $P_1$  &  $P_2$  = Load sheared by bar 1 & 2 $\sigma_1$  &  $\sigma_2$  = Stress sheared induced in bar 1 & 2

Now the total load on the composite bar

is equal to the sum of the load carried by the two bars.

$$\therefore P = P_1 + P_2 \quad \text{--- } \textcircled{1}$$

the stress in bar 1

$$\sigma_1 = \frac{P_1}{A_1} \quad (\text{or}) \quad P_1 = \sigma_1 A_1 \quad \text{--- } \textcircled{2}$$

the stress in bar 2

$$\sigma_2 = \frac{P_2}{A_2} \quad (\text{or}) \quad P_2 = \sigma_2 A_2 \quad \text{--- } \textcircled{3}$$

Substitute the values in equation  $\textcircled{1}$

we get

$$P = \sigma_1 A_1 + \sigma_2 A_2$$

since the ends of the two bars are rigidly connected, each bar will change in length by the same amount.

$$\therefore \text{strain in bar 1} \Rightarrow e_1 = \frac{\sigma_1}{E_1}$$

$$\text{Similarly strain in bar 2} \Rightarrow e_2 = \frac{\sigma_2}{E_2}$$

But strain in bar 1 = strain in bar 2

$$\therefore \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

from the above equation the stresses  $\sigma_1$  &  $\sigma_2$  can be determined.

(12) Modulus Ratio: the ratio of  $\frac{E_1}{E_2}$  is called

the modulus ratio of the first material to the second.

Temperature stresses:-

Thermal stresses are the stresses induced

in a body due to change

in temperature. The body

is allowed to expand or contract freely.

Consider a body which is heated to a certain temperature.

Let  $L$  = original length of the body

$T$  = Rise in temperature

$E$  = Young's modulus

$\alpha$  = co-efficient of linear expansion

$\Delta L$  = extension of rod due to rise of temperature.

i) If the road is free to expand, then extension of the rod is given by

$$\boxed{\Delta L = \alpha \cdot T \cdot L}$$

then compressive strain =  $\frac{\text{Decrease in length}}{\text{Original length}}$

$$= \frac{\alpha \cdot T \cdot L}{L + \alpha \cdot T \cdot L}$$

$$= \frac{2TL}{L}$$

$$\text{Stress} \approx \frac{\alpha \Delta T L}{L}$$

$$\approx \alpha \cdot \Delta T$$

$$\text{But } \frac{\text{Stress}}{\text{Strain}} = E$$

$$\text{Strain}$$

$$\text{Stress} = E \times \text{Strain}$$

$$\therefore \text{Stress} = E \times \alpha \cdot \Delta T$$

and Load on the rod = Stress  $\times$  Area

$$P = \alpha \cdot \Delta T \cdot E \times A$$

ii) If the ends of body were fixed to rigid supports so that its expansion is prevented.

$$\therefore \text{Thermal strain } e = \frac{\text{extension prevented}}{\text{original length}}$$

$$e = \frac{\Delta L}{L} = \frac{\alpha \Delta T \cdot L}{L} = \alpha \cdot \Delta T$$

and thermal stress  $\sigma = \text{Thermal strain} \times E$

$$\sigma = \alpha \cdot \Delta T \cdot E$$

iii) If the supports yield by an amount equal to  $\delta$  then the actual expansion

$$\Delta = \alpha \Delta T L - \delta$$

$$\text{and actual stress } \frac{PL}{AE} = \alpha \Delta T L - \delta$$

$$PL = (\alpha \Delta T L - \delta) AE$$

$$P = \frac{(\alpha \Delta T L - \delta)}{L} A \cdot E$$

Note:

\* thermal stress when bars are in series



$$(CL \alpha \Delta T)_s + (CL \alpha \Delta T)_2 = \left( \frac{PL}{AE_1} \right)_s + \left( \frac{PL}{AE_2} \right)_s$$

\* parallel

$$(CL \alpha \Delta T)_s - \left( \frac{PL}{AE} \right)_s = (CL \alpha \Delta T)_s + \left( \frac{PL}{AE} \right)_s$$

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Strain Energy:— whenever a body is strained, the energy is absorbed in the body. the energy, which is absorbed in the body due to straining effect is known as strain energy. the straining effect may be due to gradually applied load or suddenly applied load (or) load with impact. the following terms will be defined.

Resilience:— the total strain energy stored in a body is commonly known as resilience. whenever the straining force is removed from the strained body the body is capable of doing work. Hence the resilience is also defined as the capacity of a strained body for doing work on the removal of the force.

Proof resilience:— the maximum strain energy stored in a body is known as proof resilience. the strain energy stored in the body will be maximum when the body is stressed upto elastic limit.

Hence, the proof resilience is the quantity of strain energy stored in a body when strained upto elastic limit.

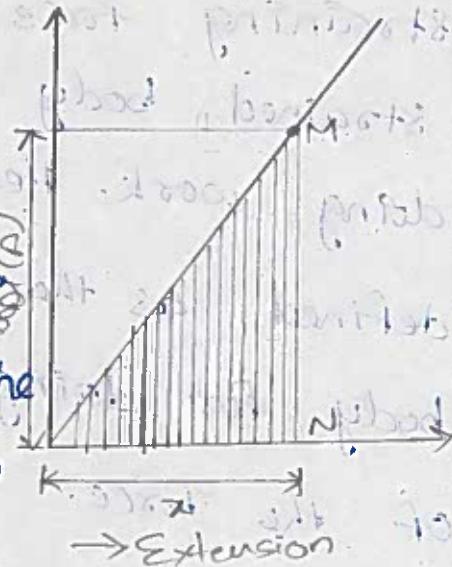
Modulus of resilience - It is defined as the proof resilience of a material per unit volume. It is an important property of a material, mathematically

$$\text{modulus of resilience} = \frac{\text{proof resilience}}{\text{volume of the body}}$$

Expression for strain energy stored in a body when the load is applied gradually:-

Fig shows load extension diagram of a body under tensile test upto elastic limit. The tensile load

'P' increase gradually from zero to the value of  $P_{(A)}$  and the extension of the body increase from zero to the value of  $x$ .



The load 'P' performs work in stretching the body. This work will be stored in the body as strain energy which is recoverable after the load P is removed

let  $P$  = Gradually applied load

$x$  = Extension of the body.

$A$  = cross-sectional area

$L$  = length of the body.

$V$  = volume of the body

$E$  = young's modulus

$U$  = strain energy stored in the body.

$\sigma$  = stress induced in the body

strain energy stored in the bar ( $U$ ) = work

done by the load Now work done = Area of triangle ONM

$$W = \frac{1}{2} \times P \times \chi \quad \text{--- (1)}$$

But load  $P = \text{stress} \times \text{Area}$

and extension  $\chi = \text{strain} \times \text{length}$

$$\chi = \frac{\text{stress} \times L}{E}$$

$$\chi = \frac{\sigma}{E} \times L$$

Substitute the values of  $P$  &  $\chi$  in

equation (1)

$\therefore$  work done by the load =  $\frac{1}{2} \times P \times \chi$

$$\text{Work done} = \frac{1}{2} \times \sigma \times A \times \frac{\sigma}{E} \times L$$

$$= \frac{1}{2} \times \sigma^2 \frac{A \cdot L}{E}$$

$$= \frac{1}{2} \times \frac{\sigma^2 V}{E}$$

$$W = \frac{\sigma^2 V}{2E}$$

( $\because$  volume  
 $V = A \times L$ )

But the work done by the load in stretching the body is equal to strain energy stored in the body.

$\therefore$  Energy stored in the body

$$U = \frac{\sigma^2}{2E} \times V$$

Expression for strain energy stored in a body when the load is applied suddenly:

when the load is applied suddenly to a body, the load is constant throughout the process of the deformation of the body. Consider a bar subjected to a sudden load

Let  $p$  = load applied suddenly

$L$  = length of the bar

$A$  = Area of the cross-section

$V = \text{volume of the bar} = (A \times L)$

$E$  = Young's modulus

$x$  = Extension of the bar

(v)  $\sigma$  = stress induced by the suddenly applied load

$V$  = strain energy stored.

AS the load is applied suddenly, the load  $P$  is constant when the extension of the bar takes place.

$\therefore$  work done by the load = load  $\times$  extension

$$W = P \cdot x \quad \text{--- (1)}$$

the maximum strain energy stored.

$$U = \frac{\sigma^2}{2E} \times \text{volume of the body}$$

$$U = \frac{\sigma^2}{2E} \times A \times L \quad \text{--- (2)} \quad [ \because V = A \times L ]$$

equating (1) = (2)

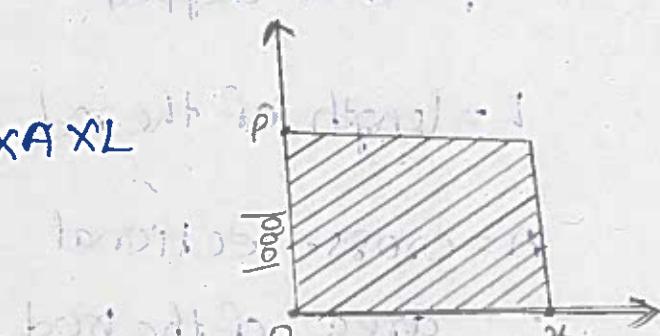
$$\left[ \because x = \frac{\sigma}{E} \times L \right]$$

$$P \cdot x = \frac{\sigma^2}{2E} \times A \times L$$

$$P \cdot \frac{\sigma}{E} \times L = \frac{\sigma^2}{2E} \times A \times L$$

$$P = \frac{\sigma \times A}{2}$$

$$\sigma = \frac{2P}{A}$$



from the above equation it is clear that the maximum stress induced due to suddenly applied load is twice the stress induced when the same load is applied gradually.

Expression for strain energy stored in a body when the load is applied with impact / shock :-

The load dropped from a certain height before the load commences to stretch the bar is a case of a load applied with impact. Consider a vertical rod fixed at the upper end and having a collar at the lower end as shown in fig.

Let the load be dropped from a height on the collar. Due to this impact there will be some extension in the rod.

Let  $p$  = load dropped

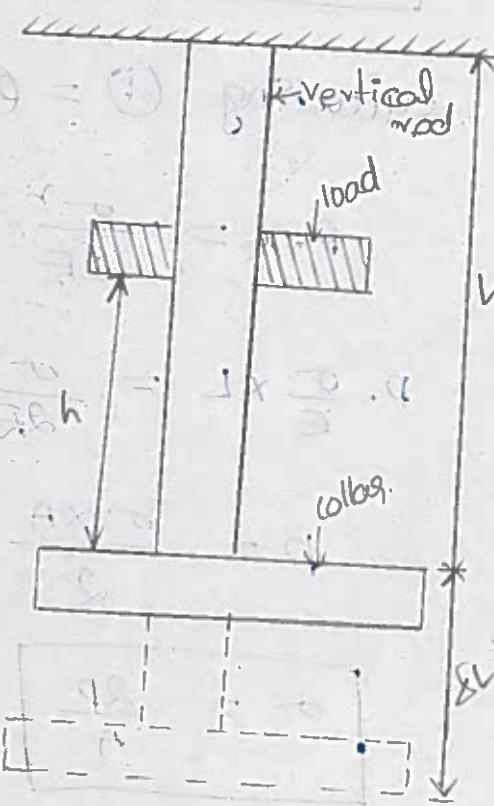
$L$  = length of the rod

$A$  = cross-sectional area of the rod

Volume ( $V$ ) =  $A \times L$

$h$  = height through

which load is dropped.



(16)  $\delta L$  = extension of the rod

$E$  = modulus of elasticity

$\sigma$  = stress induced in the rod.

the strain in the bar is given by

$$\text{strain} = \frac{\text{stress}}{E}$$

$$\frac{\delta L}{L} = \frac{\sigma}{E}$$

$$\boxed{\delta L = \frac{\sigma}{E} \times L}$$

work done by the load = load  $\times$  distance

moved  
 $P(h + \delta L)$  → ①

the strain energy stored by the rod

$$U = \frac{\sigma^2}{2E} \times V$$

$$U = \frac{\sigma^2}{2E} \times (A \times L) \rightarrow ②$$

equating work done = strain energy stored

(ie ① = ②)

$$P(h + \delta L) = \frac{\sigma^2}{2E} (A \times L)$$

$$P\left(h + \frac{\sigma}{E} L\right) = \frac{\sigma^2}{2E} \times A \times L$$

$$Ph + P \frac{\sigma}{E} L = \frac{\sigma^2}{2E} AL$$

$$\frac{\sigma^2}{2E} AL - P \frac{\sigma}{E} \cdot L - Ph = 0$$

Multiplying by  $\frac{2E}{AL}$  to both sides we get

$$\sigma^2 - P \frac{\sigma}{E} L \times \frac{2E}{2L} - Ph \frac{2E}{AL} = 0$$

$$\sigma^2 - \frac{2P}{A} \sigma - \frac{2PEh}{AL} = 0$$

The above equation is a quadratic equation in ' $\sigma$ '

$$\therefore \sigma = \frac{\frac{2P}{A} \pm \sqrt{\left(\frac{2P}{A}\right)^2 + 4 \frac{2PEh}{AL}}}{2 \times 1}$$

$\therefore \text{note}$   
 $= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{P}{A} \pm \sqrt{\frac{4P^2}{4A^2} + \frac{8PEh}{4AL}}$$

$$= \frac{P}{A} + \sqrt{\left(\frac{P}{A}\right)^2 + \frac{2PEh}{AL}}$$

$\boxed{\text{Neglected } (-) \text{ root}}$

$$= \frac{P}{A} + \frac{P}{A} \sqrt{1 + \frac{2PEh}{AL} \times \frac{A^2}{D^2}}$$

$$= \frac{P}{A} + \frac{P}{A} \sqrt{1 + \frac{2AEh}{P.L}}$$

$$\sigma = \frac{P}{A} \left[ 1 + \sqrt{1 + \frac{2AEh}{P.L}} \right] \quad \textcircled{3}$$

After knowing the value of ' $\sigma$ ' the strain energy can be obtained.

(12) Conclusion: If  $\delta L$  is very small in comparison with 'h' the work done by load  $= ph$ .

Equating the work done = strain energy stored.

$$P \cdot h = \frac{\sigma^2}{2E} AL$$

$$\sigma^2 = \frac{2E \cdot ph}{AL}$$

$$\boxed{\sigma = \sqrt{\frac{2Eph}{AL}}}$$

from the equation (8) if  $h \approx 0$  we get

$$\sigma = \frac{P}{A} [1 + \sqrt{1+0}]$$

$$\sigma = \frac{P}{A} (1+1)$$

$$\boxed{\sigma = \frac{2P}{A}}$$

which is the case of suddenly applied load.

Saint-Venant's principle :- (1855)

It was named after a french elasticity theorist "the difference b/w the effects of two different but statically equivalent loads becomes very small at sufficiently large distances from load".